

THERMAL PHYSICS

IS A HEAT WAVE A HEAT WAVE?

Expected learning outcome

Understanding how to derive the heat equation, its meaning and implications.

lecture notes



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scan the QR code or
check the course webpage

What did we learn so far?

$$\Delta U = Q + W$$

$$dS \geq \frac{\delta Q}{T}$$

$$\vec{J} = \frac{\Delta Q}{\Delta t} \propto -\vec{\nabla} T$$

lecture notes



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What did we learn so far?

$$dU = \delta Q$$

$$J_x = \frac{\delta Q}{dt} = -k \frac{dT}{dx}$$

lecture notes

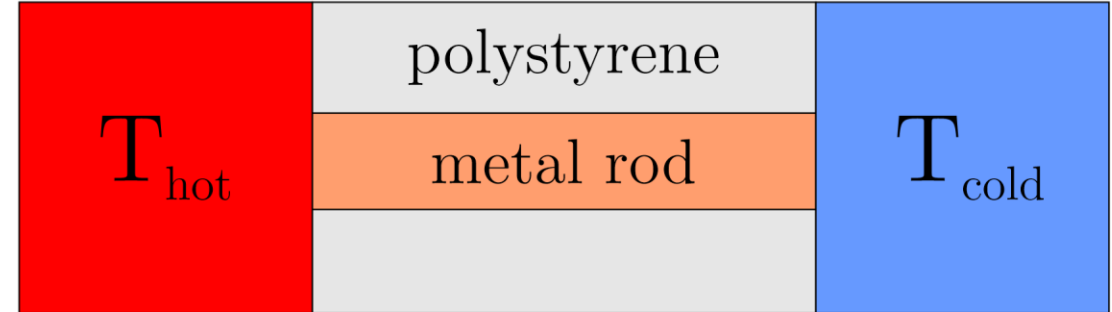


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One-dimensional heat flow in a metal rod

$$dU = \delta Q$$

$$J_x = \frac{\delta Q}{dt} = -k \frac{dT}{dx}$$



- Specific heat (c), thermal conductivity (k) and density (ρ) of the material constant and uniform.
- No heat source or sink in the system.
- Perfectly insulated, long thin bar ($L^2 \gg A$).

Net heat balance in the rod

$$dU = \delta Q$$

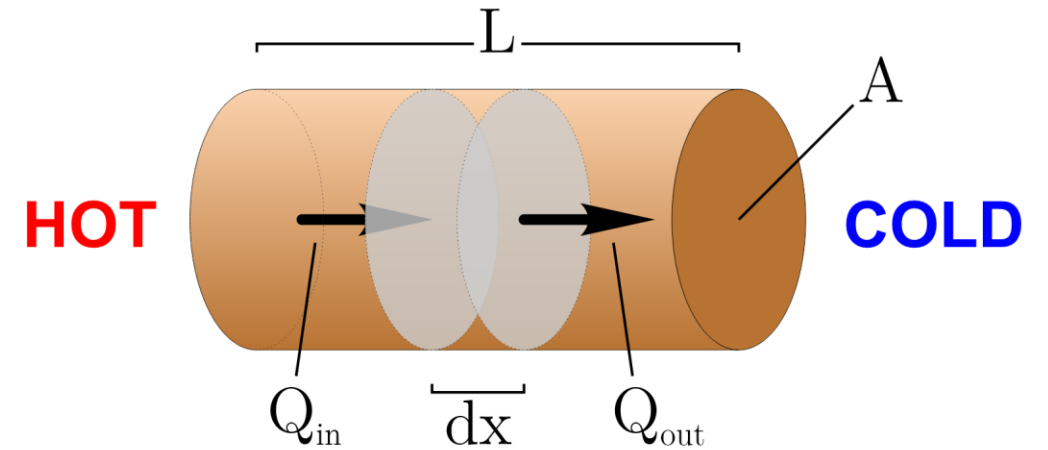
$$J_x = \frac{\delta Q}{dt} = -k \frac{dT}{dx}$$

$$\delta Q = dt \left(-k \frac{dT}{dx} \right)$$

$$\delta Q = Q_{in} - Q_{out} = dt \left(-k \frac{\partial T}{\partial x} \Big|_{x=a} + k \frac{\partial T}{\partial x} \Big|_{x=a+dx} \right)$$

thermal conductivity
thermal conductance

$$\delta Q = \kappa A \left(-\frac{\partial T}{\partial x} \Big|_{x=a} + \frac{\partial T}{\partial x} \Big|_{x=a+dx} \right) dt$$

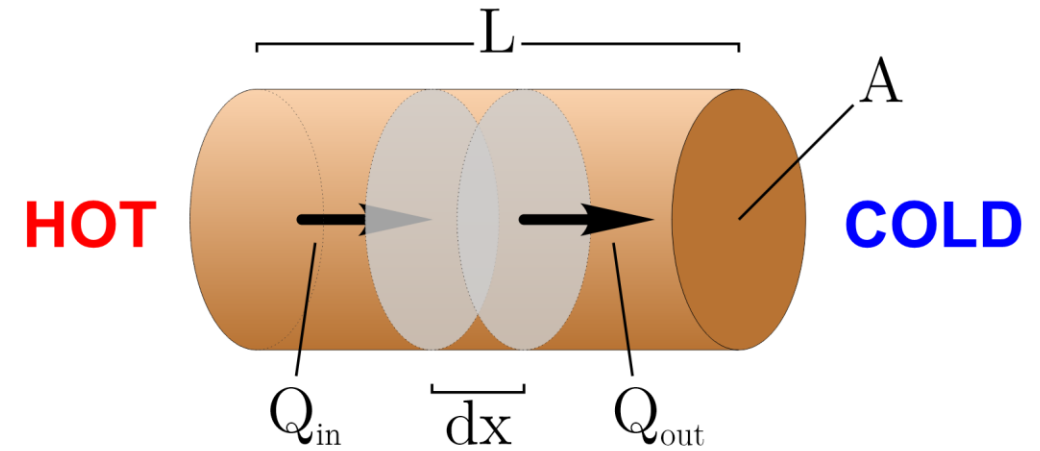


Net balance of thermal energy

should I stay or should I go?

$$dU = \delta Q$$

$$\delta Q = \kappa A \left(-\frac{\partial T}{\partial x} \bigg|_{x=a} + \frac{\partial T}{\partial x} \bigg|_{x=a+dx} \right) dt$$



$$dU = \overbrace{c\rho A dx}^{\text{mass}} T(x, t + dt) - \overbrace{c\rho A dx}^{\text{infinitesimal volume element}} T(x, t)$$

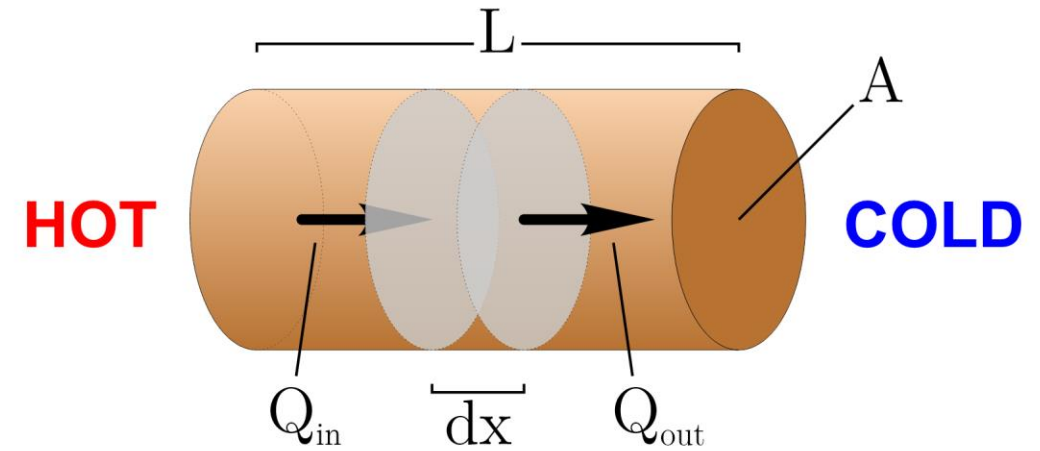
heat capacity of the volume element $A dx$

$$\delta Q = c\rho A dx [T(x, t + dt) - T(x, t)]$$

The heat equation: how thermal energy redistribute in space and time

$$\delta Q = c\rho A dx [T(x, t + dt) - T(x, t)]$$

$$\delta Q = \kappa A \left(-\frac{\partial T}{\partial x} \Big|_{x=a} + \frac{\partial T}{\partial x} \Big|_{x=a+dx} \right) dt$$



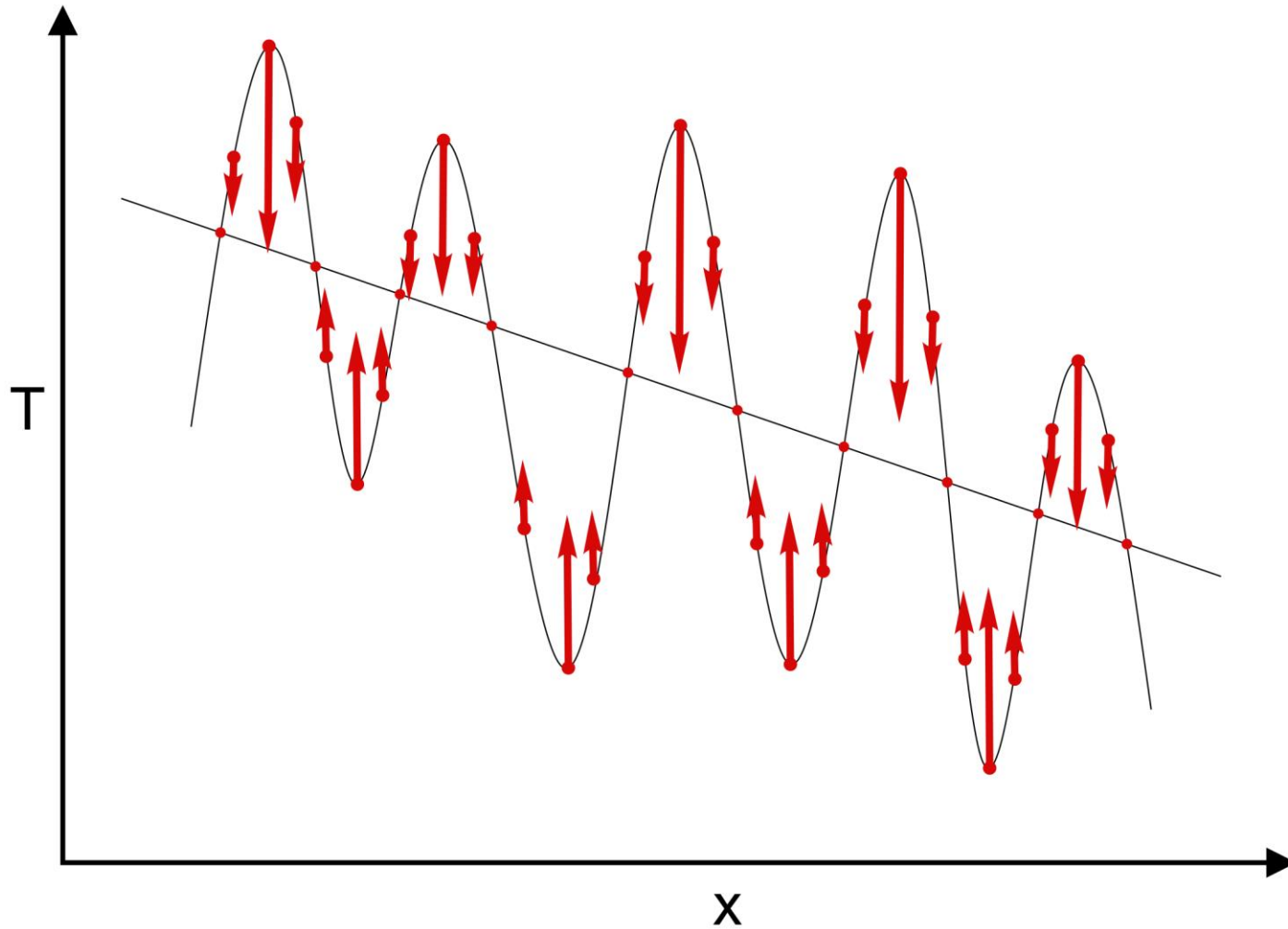
$$c\rho A dx [T(x, t + dt) - T(x, t)] = \kappa A dt \left(\frac{\partial T}{\partial x} \Big|_{x=a+dx} - \frac{\partial T}{\partial x} \Big|_{x=a} \right)$$

$$\frac{T(x, t + dt) - T(x, t)}{dt} = \frac{\kappa}{c\rho} \frac{\frac{\partial T}{\partial x} \Big|_{x=a+dx} - \frac{\partial T}{\partial x} \Big|_{x=a}}{dx}$$

thermal diffusivity

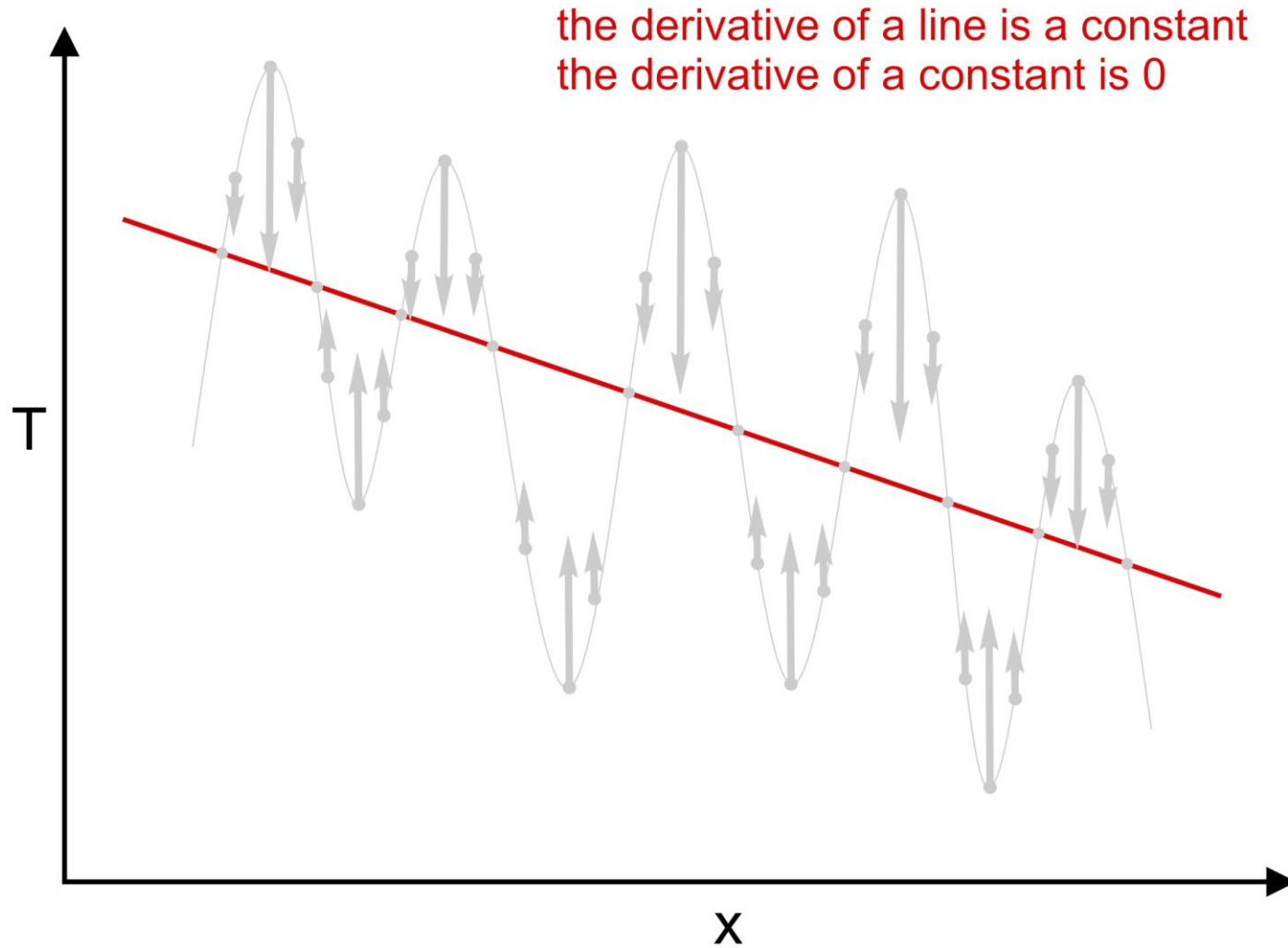
$$\frac{\partial T}{\partial t} = \alpha \frac{\partial^2 T}{\partial x^2}$$

Heat flows to homogenize temperature in the rod (2nd law)



$$\frac{\partial T}{\partial t} = \alpha \frac{\partial^2 T}{\partial x^2}$$

...until steady state is reached



$$\frac{\partial T}{\partial t} = \alpha \frac{\partial^2 T}{\partial x^2}$$

Is a heat wave a heat wave?

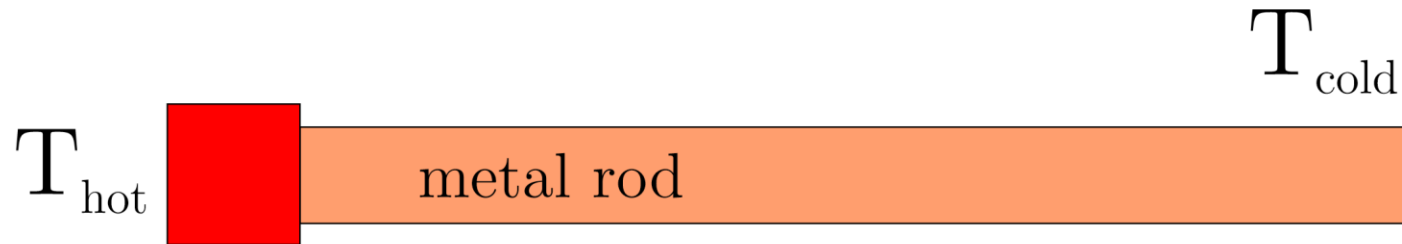
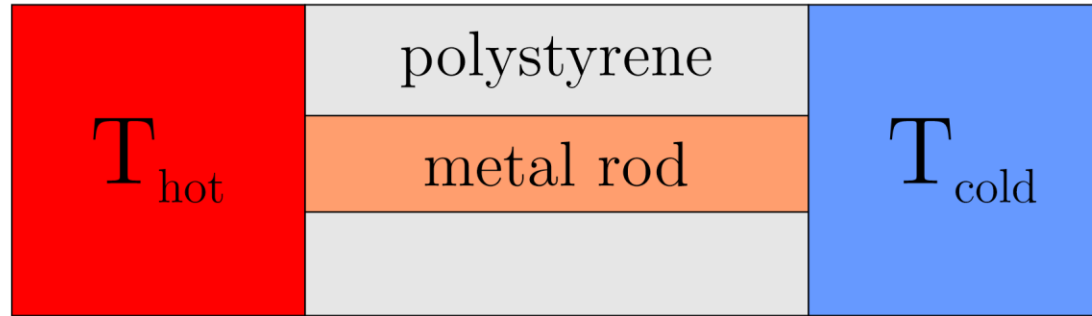
$$\frac{\partial T}{\partial t} = \alpha \frac{\partial^2 T}{\partial x^2}$$

$$\frac{\partial^2 u}{\partial t^2} = v^2 \frac{\partial^2 u}{\partial x^2}$$

$$\frac{\partial \phi}{\partial t} = D \frac{\partial^2 \phi}{\partial x^2}$$



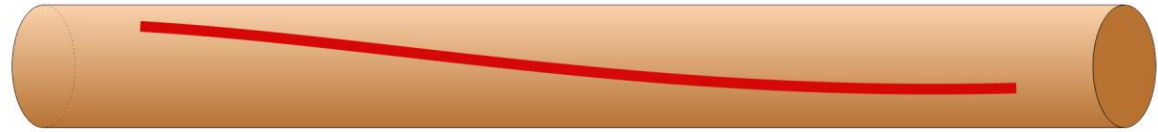
Is a heat wave a heat wave? Brief demonstration



Is a heat wave a heat wave?

$$\frac{\partial T}{\partial t} = \alpha \frac{\partial^2 T}{\partial x^2}$$

irreversible dissipative process - diffusion



$$\frac{\partial^2 u}{\partial t^2} = v^2 \frac{\partial^2 u}{\partial x^2}$$

stress wave - electrical signals

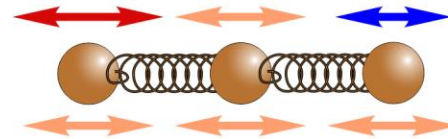


$$\frac{\partial \phi}{\partial t} = D \frac{\partial^2 \phi}{\partial x^2}$$



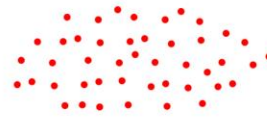
... but copper atoms do not diffuse!?

phonons (lattice vibrations)

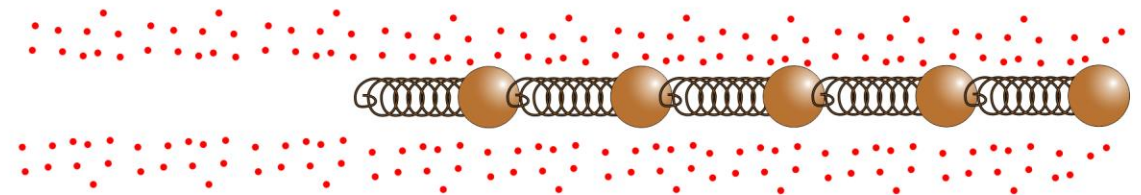
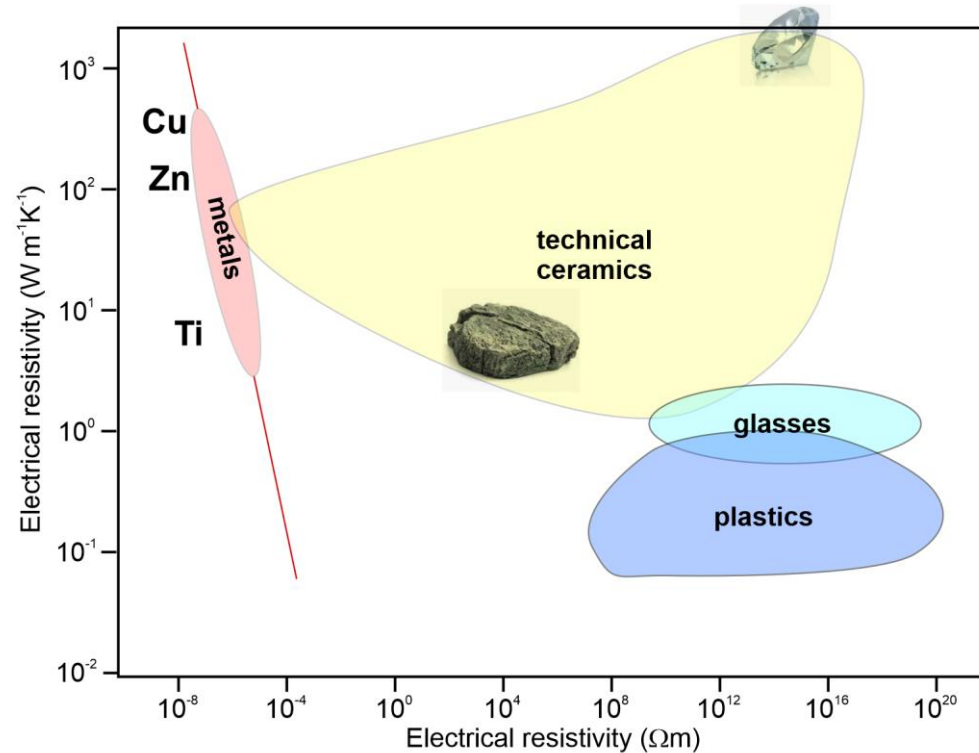


Einstein and Debye models for heat capacity
A gas of phonons

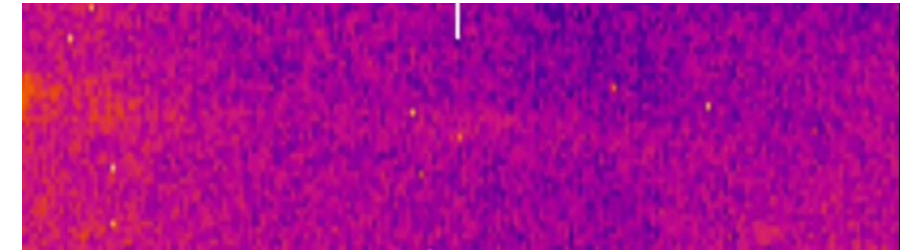
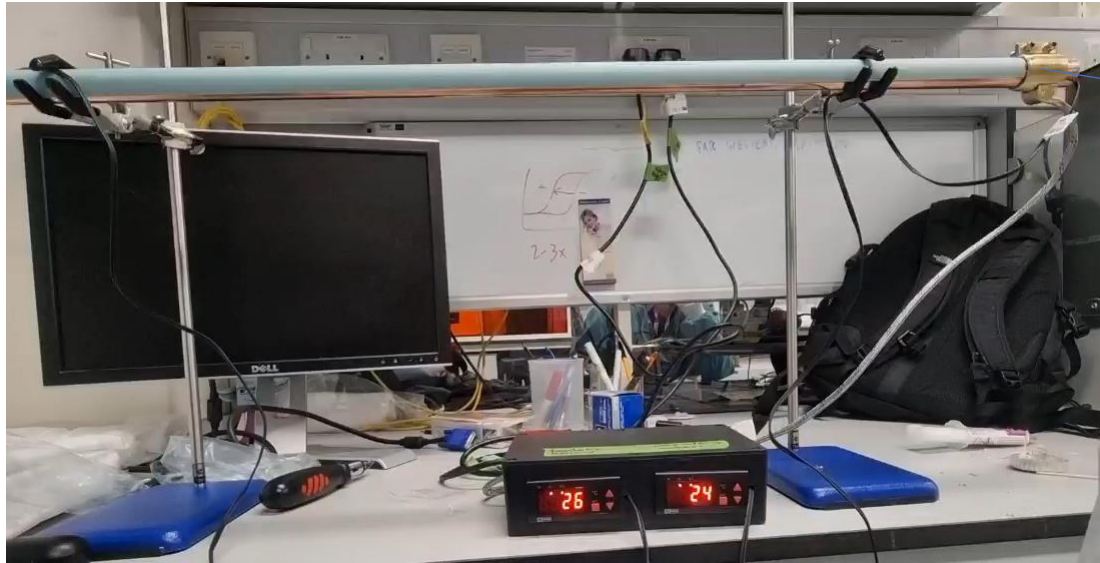
free electrons (metals)



Drude (classical, Wiedemann-Franz law)
Drude-Sommerfeld (Fermi gas)
A gas of fermions



Next



How do we solve the heat equation?

Newton's law of cooling

How do we measure thermal conductance?
Frequency and time domain methods.

